Influence of potential fluctuations on the instrumental functions of electrostatic analyzers

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An equation which relates the output signal of an electrostatic dispersion analyzer and the energy distribution function of the charged particles entering it is derived with the fluctuations of the potentials on the defecting electrodes taken into account. Solutions of this equation are obtained. The influence of noise on the instrumental functions of analyzers is considered.
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INTRODUCTION

It was shown in the preceding paper[1] that the energy distribution function of the charged particles at the entrance to an electrostatic analyzer and the output signal of the analyzer are related by the expression

$$I(U_1,\ldots,U_n)=\int_0^{+\infty} f(E)A(U_1/E,\ldots,U_n/E)dE, \quad (1)$$

where $A(U_1/E,\ldots,U_n/E)$ is the instrumental function of the analyzer; $U_1,\ldots,U_n$ are the potentials on the analyzer electrodes; and $f(E)$ is the energy distribution function of the charged particles.

Relation (1) also holds when the stray fields arising from the actual geometry of the analyzer are taken into account. However, the form of the instrumental function of a charged-particle analyzer is determined not only by its geometry, but also by the fluctuations of the fields within the analyzer, i.e., by the noise. If fluctuations whose characteristic period is much greater than the time of flight of a particle in the analyzer are considered, it can be assumed that the tuning energy of the analyzer fluctuates.

In this paper we shall consider the influence of noise on the instrumental function of an electrostatic dispersion analyzer.

INFLUENCE OF FLUCTUATIONS ON THE INSTRUMENTAL FUNCTION OF AN ANALYZER WITH ONE DEFLECTING ELECTRODE

Let us assume for an electrostatic analyzer, in which only one electrode is under a potential, that the voltage $\check{U}$ on the analyzer electrode fluctuates about the mean potential U with a distribution function $\varphi(\check{U},U)$, which satisfies the following conditions

$$\int_{-\infty}^{+\infty}\varphi(\check{U},U)d\check{U}=1,$$

$$U=\langle\check{U}\rangle=\int_{-\infty}^{+\infty}\check{U}\cdot\varphi(\check{U},U)dU,$$

$$\sigma^2(\check{U})=\int_{-\infty}^{+\infty}(\check{U}-U)^2\varphi(\check{U},U)dU. \quad (2)$$

If the measurement time of the particle current at the analyzer exit is much greater than the characteristic period of the fluctuations, then, using Eq. (1), we can represent the mean value of the current at the analyzer exit in the form

$$I(U)=\int_{-\infty}^{+\infty}\varphi(\check{U},U)\tilde{I}(\check{U})d\check{U}$$

$$=I_0\int_{-\infty}^{+\infty}\varphi(\check{U},U)\int_0^{+\infty}f(E)A\left(\frac{q\check{U}}{E}\right)dEd\check{U} \quad (3)$$

or

$$I(U)=I_0\int_0^{+\infty}f(E)B(U,\,E)dE, \quad (4)$$

where

$$B(U,E)=\int_{-\infty}^{+\infty}\varphi(\check{U},U)A(q\check{U}/E)d\check{U} \quad (5)$$

is the instrumental function of the analyzer with consideration of the noise.

Let us find an approximate solution of this equation. Expanding $\tilde{I}(\check{U})$ into a Taylor series about the point U, assuming that $\tilde{I}(U)\approx I(U)$ in a first approximation, and taking into account the terms of the series containing derivatives no higher than the second, from (3) we obtain the expression

$$\tilde{I}(U)\approx I(U)-\frac{\sigma^2}{2}\frac{d^2I}{dU^2}. \quad (6)$$

A solution of the equation

$$\tilde{I}(U)=I_0\int_0^{+\infty}f(E)A\left(\frac{qU}{E}\right)dE \quad (7)$$

can be obtained in the form of a series:[1]

$$f(kU)=\sum_{n=0}^{+\infty}B_nU^{n-1}\frac{d\tilde{I}^n(U)}{dU^n}, \quad (8)$$

where

$$B_0=\frac{1}{kC_{00}},\quad B_n=-\frac{1}{C_{n0}}\sum_{i=0}^{n}B_i\frac{C_{i(n-i)}}{(n-i)!}.$$

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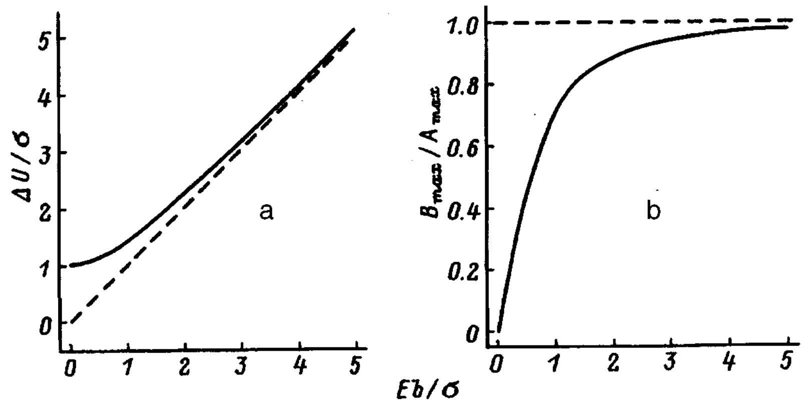

FIG. 1. Curves for an electrostatic dispersion analyzer: a—The ratio between the standard deviation of the instrumental function ΔU (with allowance for the influence of the fluctuations of the potential) and the standard deviation σ of the noise and, b—the ratio of the height of the instrumental function B_{max} (with consideration of the influence of the potential fluctuations) to the height of the instrumental function A_{max} (without consideration of the noise) as functions of the ratio of the standard deviation of the instrumental function (without consideration of the noise) bE to the standard deviation σ of the fluctuations of the potentials on the analyzer electrodes.

Ultimately, to within the terms with a second derivative, for the ''true distribution'' as a function of the output signal $I(U)$ we obtain

$$f(kU)\approx\frac{i(U)}{C_{10}I_0U}-\frac{Ud^2I/dU^2}{2C_{10}I_0}\left(1-\frac{C_{10}^2}{C_{00}C_{20}}+\frac{\sigma^2}{U^2}\right), \quad (9)$$

where the $C_{nm}=\int_0^{+\infty}z^{n-1}(z-k)^mA(q/z)dz$ are constants.

It is expedient to select the analyzer constant such that in the range of energies where the influence of the noise is negligibly small, the analyzer constant would be determined from the expression[1]

$$k=C_{10}/C_{00}. \quad (10)$$

Let us now consider the influence of the noise on the form of the instrumental function of the analyzer. For an arbitrary noise distribution function $\varphi(\check{U},U)$, an approximate analytical expression for the instrumental function $B(U,E)$ of the analyzer can be obtained in two limiting cases: in the range of energies where the width of the instrumental function is determined mainly by the noise and in the range of energies where the influence of the noise on the instrumental function is weak.

In the former case, applying the theorem of the mean[2] to the integral in Eq. (5) and assuming that $A(q/z)\equiv 0$ for $z\leqslant 0$, we obtain

$$B(U,E)=C_{00}\hat{U}\varphi(\hat{U},U). \quad (11)$$

Assuming that $\check{U}\varphi(\check{U},U)$ varies weakly across the width of the instrumental function $A(q\check{U}/E)$, we have

$$\hat{U}\varphi(\hat{U},U)\approx\frac{E}{k}\varphi\left(\frac{E}{k},U\right). \quad (12)$$

Substituting this expression into (11), we ultimately obtain

$$B(U,E)\approx(C_{00}/k)E\varphi\left(\frac{E}{k},U\right), \quad (13)$$

i.e., the form of the instrumental function is determined mainly by the noise distribution function, and the value of the instrumental function at the maximum depends on the energy of the particles.

For example, for a noise distribution function of the form $\varphi(\check{U}-U)$ it follows from expression (13) that the transmission of the analyzer will decrease as the energy of the particles decreases. We use the term transmission to refer to a coefficient of an analyzer that is equal to the ratio of the maximum output current of the analyzer to the current of a monoenergetic beam at its entrance.

In the second case, since the noise distribution function $\varphi(\check{U},U)$ is considerably narrower than the instrumental function $A(q\check{U}/E)$, applying the theorem of the mean and assuming that $A(q\hat{U}/E)\approx A(qU/E)$, we obtain

$$B(U,\ E)\approx A\left(\frac{qU}{E}\right), \quad (14)$$

i.e., the width of the instrumental function will increase linearly with increasing energy, and the transmission of the analyzer will remain unchanged.

Let us trace how the form of the instrumental function of the analyzer varies in the presence of noise as a function of the energy in the case of a noise distribution function of the form $\varphi(\check{U},U)=\varphi(\check{U}-U)$. For this purpose we treat the instrumental function $B(U,E)$ as a distribution function with respect to U, which can be characterized by the mean $\langle U\rangle$ and the variance $\langle\Delta U^2\rangle$,

$$\langle U\rangle=\frac{\int_{-\infty}^{+\infty}UB(U,E)dU}{\int_{-\infty}^{+\infty}B(U,E)dE}, \quad (15)$$

$$\langle\Delta U^2\rangle=\frac{\int_{\infty}^{+\infty}(U-\langle U\rangle)^2B(U,E)dU}{\int_{-\infty}^{+\infty}B(U,E)dU}. \quad (16)$$

After some relatively simple mathematical manipulations, we find that the mean value of the potential is proportional to the energy:

$$\langle U\rangle=\frac{C_1}{C}E. \quad (17)$$

Here $C=\int_{-\infty}^{+\infty}A(qx)dx$, and $C_1=\int_{-\infty}^{+\infty}xA(qx)dx$, where $x=\check{U}/E$.

We calculate the variance of the instrumental function by substituting expression (17) into Eq. (16) and performing the integration:

$$\langle\Delta U^2\rangle=\sigma^2+b^2E^2. \quad (18)$$

Here $b^2=(C_2C-C_1^2)/C^2$, where $C_2=\int_{-\infty}^{+\infty}x^2A(qx)dx$.

Figure 1 shows qualitatively the behavior of the width of the instrumental function and its value at the maximum as a function of E. The expressions (13), (14), and (18) obtained allow us to conclude that in the range of energies where $E \ll \sigma/b$ the width of the instrumental function scarcely depends on the energy, and its value at the maximum increases linearly with the energy. Conversely, in the range of energies where $E \gg \sigma/b$ the width of the instrumental function increases linearly with increasing energy, and the value at the maximum remains constant. The quantity $E^2 b^2$ corresponds to the variance of the instrumental function with respect to U in the absence of potential fluctuations and can be determined experimentally.

It should be noted that a solution of Eq. (3) can be found in general form for a noise distribution function of the form under consideration $\varphi(\check{U}-U)$. Applying the Fourier transformation to Eq. (3), we obtain a solution in the form of a series[3]

$$\int_0^{+\infty} f(E) A\left(\frac{qU}{E}\right) dE = \sum_{s=0}^{+\infty} D_s \frac{d^s I(U)}{dU^s} = \tilde{I}(U), \tag{19}$$

where

$$D_0 \int_{-\infty}^{+\infty} \varphi(z) dz = 1, \quad \sum_{s=0}^{m} D_s \int_{-\infty}^{+\infty} z^{m-s} \varphi(z) dz \frac{1}{(m-s)!} = 0. \tag{20}$$

A series solution of Eq. (19) was found in Ref. 1. In the present case it has the form

$$f(kU) = \sum_{n=0}^{+\infty} B_n U^{n-1} \frac{d\tilde{I}^n(U)}{dU^n}, \tag{21}$$

where

$$B_0 = \frac{1}{kC_{00}}, \quad B_n = -\frac{1}{C_{n0}} \sum_{i=0}^{n} B_i \frac{C_{i(n-i)}}{(n-i)!}. \tag{22}$$

Ultimately, for the general solution we obtain

$$f(kU) = \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} D_m B_n U^{n-1} \frac{d^{m+n} I(U)}{dU^{m+n}}, \tag{23}$$

where the analyzer constant k is specified by expression (7).

INFLUENCE OF NOISE ON THE INSTRUMENTAL FUNCTION OF AN ANALYZER WITH TWO DEFLECTING ELECTRODES

All the arguments advanced above referred to the case in which a potential was supplied to one electrode. Let us now consider the case in which potentials are supplied to two electrodes. The expression for the mean value of the current at the analyzer exit under the condition that the measurement time is much greater than the characteristic period of the fluctuations has the form

$$\begin{aligned} I(U_1,U_2) &= I_0 \int \int_{-\infty}^{+\infty} \varphi(\check{U}_1,\check{U}_2,U_1,U_2) \\ &\quad \times \int_0^{+\infty} A\left(\frac{\check{U}_1}{E},\frac{\check{U}_2}{E}\right) f(E)\, dE\, d\check{U}_1 d\check{U}_2 \\ &= \int \int_{-\infty}^{+\infty} \varphi(\check{U}_1,\check{U}_2,U_1,U_2) \tilde{I}(\check{U}_1,\check{U}_2) d\check{U}_1 d\check{U}_2, \end{aligned} \tag{24}$$

where $\varphi(\check{U}_1,\check{U}_2,U_1,U_2)$ is the normalized distribution function of the potential fluctuations, which satisfies the conditions

$$\begin{aligned} &\int \int_{-\infty}^{+\infty} \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 = 1, \\ &\int \int_{-\infty}^{+\infty} \check{U}_1 \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 = U_1, \\ &\int \int_{-\infty}^{+\infty} \check{U}_2 \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 = U_2, \\ &\int \int_{-\infty}^{+\infty} (\check{U}_1 - U_1)^2 \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 = \sigma_1^2, \\ &\int \int_{-\infty}^{+\infty} (\check{U}_2 - U_2)^2 \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 = \sigma_2^2, \\ &\int \int_{-\infty}^{+\infty} (\check{U}_1 - U_1)(\check{U}_2 - U_2) \varphi(\check{U}_1,\check{U}_2,U_1,U_2) d\check{U}_1 d\check{U}_2 \\ &\quad = \mathrm{co}\nu(\check{U}_1,\check{U}_2). \end{aligned} \tag{25}$$

Assuming that the function $\tilde{I}(\check{U}_1,\check{U}_2)$ varies weakly across the width of the noise distribution function, expanding $\tilde{I}(\check{U}_1,\check{U}_2)$ in a Taylor series about the point (U_1,U_2), assuming that $\tilde{I}(U_1,U_2) = I(U_1,U_2)$ in a first approximation, and keeping the terms with derivatives no higher than the second order, we obtain from Eq. (24)

$$\begin{aligned} \tilde{I}(U_1,U_2) \approx I(U_1,U_2) - \frac{1}{2}\Bigg(&\sigma_1^2 \frac{\partial^2 I}{\partial U_1^2} \\ &+ 2\mathrm{co}\nu(\check{U}_1,\check{U}_2) \frac{\partial^2 I}{\partial U_1 \partial U_2} + \sigma_2^2 \frac{\partial^2 I}{\partial U_2^2}\Bigg). \end{aligned} \tag{26}$$

If the mean values of the potentials are linearly related, i.e., if $U_2 = \lambda U_1$, and if we take into account that the expression for the energy distribution function in terms of $\tilde{I}(U_1,U_2)$ has the form[1]

$$f(kU_1) = \frac{1}{I_0} \sum_{n=0}^{\infty} B_n U_1^{n-1} \frac{d\tilde{I}^n(U_1,\lambda U_1)}{dU_1^n}, \tag{27}$$

the solution of Eq. (24) to terms with a second derivative will be as follows:

$$
\begin{aligned}
f(kU_1) \approx \frac{1}{I_0}\Bigg(& \frac{I(U_1,\lambda U_1)}{C_{10}U_1} \\
& - \frac{U_1}{2}\,\frac{C_{20}C_{00}-C_{10}^2}{C_{00}C_{10}C_{20}}\,\frac{d^2 I(U_1,\lambda U_1)}{dU_1^2}\Bigg) \\
& - \frac{U_1}{2I_0}\Bigg(\sigma_1^2\frac{\partial^2 I(U_1,U_2)}{\partial U_1^2} \\
& + 2\operatorname{co}\nu(\check{U}_1,\check{U}_2)\frac{\partial^2 I(U_1,U_2)}{\partial U_1\partial U_2} \\
& + \sigma_2^2\frac{\partial^2 I(U_1,U_2)}{\partial U_2^2}\Bigg)\Bigg|_{U_2=\lambda U_1}. \qquad (28)
\end{aligned}
$$

It follows from expression (28) that a more exact reconstruction of the energy spectrum with the noise corrections under the condition that the mean values of the potentials are linearly related requires knowledge of the values of the output signal not only at points where the potentials are linearly related, but also in a certain vicinity of the latter in the general case. If the additional conditions on the potential fluctuations

$$
\sigma_2^2=\lambda^2\sigma_1^2, \quad \operatorname{co}\nu(\check{U}_1,\check{U}_2)=\lambda\sigma_1^2 \qquad (29)
$$

are satisfied, expression (28) transforms into an expression in total derivatives, and to reconstruct such a spectrum to within the corrections associated with a second derivative, it is sufficient to know the signal at the points where the mean values of the potentials are linearly related.

We note that the results obtained for spectrometers with two electrodes can easily be generalized to the case of an analyzer with n electrodes.

CONCLUSIONS

In conclusion, let us briefly describe the results obtained.

1. Equations (3) and (24) make it possible to describe the relationship between the energy distribution function of the charged particles and the output signal of an electrostatic analyzer operating in the spectrometer mode with the influence of fluctuations of the potentials on its electrodes taken into account.

2. The approximate solutions (9) and (28) of these equations for an arbitrary distribution function of the potential fluctuations permit taking into account the corrections associated with noise to the reconstructed energy distribution.

3. For analyzers with several electrodes under different potentials, it is not enough to know the values of the output signal at the points where the mean values of the potentials are linearly related. More exact reconstruction of the true energy distribution requires knowledge of the values of the output signal in the vicinity of these points and the dispersion of the noise.

4. An examination of the behavior of the instrumental function as a function of various parameters in the case of an analyzer with one deflecting electrode shows that at energies for which the width of the instrumental function of the analyzer is determined mainly by the potential fluctuations, the transmission of the analyzer will fall off as the energy decreases.

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[3] V. V. Raznikov and M. O. Raznikova, *Analytical Mass Spectrometry* [in Russian], Nauka, Moscow (1992).

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